ENGINEERING PHYSICS

UNIT - 4

LASERS

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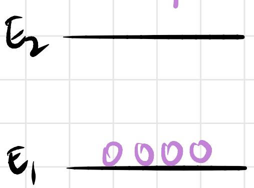
LASER

- Light Amplification by Stimulated Emission of Radiation (Einsten's idea)
- Took ~50 years to design MASER (microwave)
- Only then, LASER (gas laser)

Radiation Interacting with Matter
- Even in BBR
- In thermal equilibrium, 3 processes

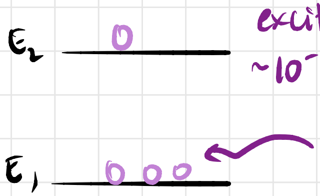
(1) Induced absorption



$N_1 > N_2$

$E_2 - E_1 = h\nu$

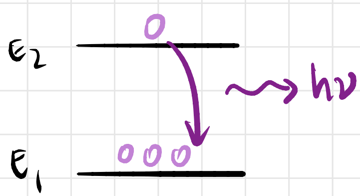
if $h\nu$ provided from radiation



excited atom
~10^{-9} s (unstable)

(2) Spontaneous emission

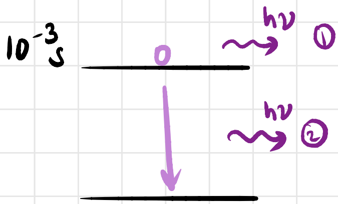
returns back to ground state spontaneously



ground state atom

homework

(3) Stimulated Emission



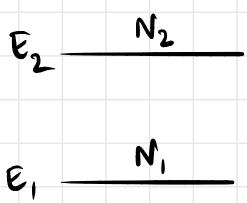
same energy and momentum direction] amplified system

- coherent, monochromatic
- if the excited states have longer lifetimes of the order of 10^{-3} s, such states are called metastable states
- will not spontaneously de-excite; requires photon of energy $E = E_2 - E_1 = h\nu$ in vicinity to stimulate emission

Einstein's Energy Density Expression - Einstein Coefficients

Let us consider an atomic system with only 2 energy levels, E_1 and E_2

Let N_1 and N_2 be the populations of E_1 and E_2



Let us supply energy density U_ν to the system (radiation)

Under normal conditions, $N_1 > N_2$

(1) Induced Absorption

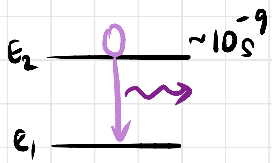
rate of induced absorption $\propto$ (i) no. of atoms in level 1 (N_1)
(ii) energy density (U_ν)

$$\propto N_1 U_\nu$$

$$R_{abs} = B_{12} N_1 U_\nu \quad \text{——} \quad (1)$$

where B_{12} is called Einsten's coefficient of induced absorption

(2) Spontaneous Emission



rate of spontaneous emission $\propto$ no. of atoms in level 2 (N_2)

$$\propto N_2$$

$$R_{spon} = A_{21} N_2 \quad \text{——} \quad (2)$$

where A_{21} is called Einsten's coefficient of spontaneous emission

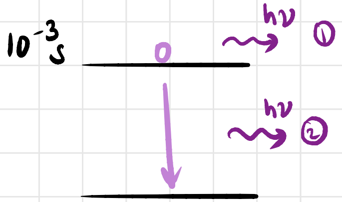
Photons are emitted in random directions

(3) Stimulated emission

pertobation + evry

when a photon of energy $h\nu = E_2 - E_1$ is in the vicinity of an excited atom (E_2), it is vulnerable to de-excitation

It de-excites and emits a photon of $E = h\nu$



① & ② photons are perfectly coherent

(phase, ν, E, k, dir same)

chain rxn occurs

(if the energy state is a metastable state)

rate of stimulated emission $\propto$ (i) no. of atoms in level 2 (N_2)
(ii) energy density (U_ν)

$$\propto N_2 U_\nu$$

$$= B_{21} N_2 U_\nu \qquad \text{——— (3)}$$

where B_{21} is called Einsten's coefficient of stimulated emission

At thermal equilibrium

rate of absorption = rate of emission

$$B_{12} N_1 U_\nu = B_{21} N_2 U_\nu + A_{21} N_2$$

$$U_\nu = \frac{A_{21} N_2}{(B_{12} N_1 - B_{21} N_2)}$$

$$U_\nu = \frac{A_{21} \cancel{N_2}}{B_{21} \cancel{N_2}\left(\frac{B_{12}}{B_{21}}\frac{N_1}{N_2} - 1\right)} = \frac{A_{21}/B_{21}}{(B_{12}/B_{21})\frac{N_1}{N_2} - 1}$$

Boltzmann equation

$$\frac{N_1}{N_2} = e^{\frac{E_2 - E_1}{kT}}$$

$$U_\nu = \frac{\left(\frac{A_{21}}{B_{21}}\right)}{\left(\frac{B_{12}}{B_{21}}\right) e^{\frac{h\nu}{kT}} - 1}$$

— Einstein's energy density expression

Compare Einstein's energy density expression with Planck's energy density expression.

Einstein's expression

$$U_\nu = \frac{\left(\frac{A_{21}}{B_{21}}\right)}{\left(\frac{B_{12}}{B_{21}}\right)e^{\frac{h\nu}{kT}} - 1}$$

Planck's expression

$$U_\nu = \frac{8\pi h\nu^3}{c^3}\left(\frac{1}{e^{h\nu/kT} - 1}\right)$$

$$\frac{B_{12}}{B_{21}} = 1$$

$$B_{12} = B_{21} = B$$

Probability of rate of induced absorption is equal to probability of rate of stimulated absorption.

$A_{21} = A$, $A \propto \frac{1}{\tau}$

$$\frac{A}{B} = \frac{8\pi h\nu^3}{c^3}$$

Probability of rate of absorption $\propto \nu^3$

A & B are called Einstein coefficients

Q: An emission system has 2 levels giving raise to an emission $\lambda = 546.1$ nm (green). If the population of the lower state is 4×10^{22} at 600K, estimate the population of higher energy state

$$\frac{N_1}{N_2} = e^{\frac{h\nu}{kT}} = e^{\frac{hc}{\lambda kT}} = e^{43.91}$$

$$\frac{N_1}{N_2} = 1.175 \times 10^{19}$$

$$N_2 = 3403.3$$

Q: The ratio of population of higher energy state to lower energy state is 5×10^{-19} at T = 400K Find emission λ and $\frac{A}{B}$.

homework: doubt

↓

? hap to do

$$\frac{N_2}{N_1} = 5 \times 10^{-19} = e^{\frac{-hc}{\lambda kT}}$$

$$\frac{hc}{\lambda kT} = 42.1397$$

$$\lambda = 853.6 \text{ nm}$$

$$\nu = 3.512 \times 10^{14}$$

$$\frac{A}{B} = \frac{8\pi h \nu^3}{c^3}$$

$$= 2.68 \times 10^{-14}$$

Q: A hypothetical atom has uniformly-separated energy levels at a separation of 1.2eV. Find the ratio of no. of atoms in 7^{th} excited state to that of the 5^{th} excited state at 300K.

$$\frac{N_8}{N_6} = e^{\frac{-(E_8 - E_6)}{kT}}$$

$$= e^{\frac{-2h\nu}{kT}} = e^{\frac{-2\times 1.2eV}{kT}}$$

$$\frac{N_8}{N_6} = 4.81 \times 10^{-41} \quad (5.21 \times 10^{-41})$$

Q: If R_1 = rate of stimulated emission and R_2 = rate of spontaneous emission between 2 energy levels, then show that

$$\lambda = \frac{hc}{kT \ln\left(\frac{R_2}{R_1} + 1\right)}$$

$$R_1 = B N_2 U_\nu$$
$$R_2 = A N_2$$

$$\frac{A}{B} = \frac{8\pi h\nu^3}{c^3} \longrightarrow (1)$$

$$U_\nu \frac{R_2}{R_1} = \frac{A}{B} \quad \text{———} \quad (2)$$

$$U_\nu = \frac{8\pi h\nu^3}{c^3}\left(\frac{1}{e^{h\nu/kT} - 1}\right)$$

$$\frac{8\pi h\nu^3}{c^3}\left(\frac{1}{e^{h\nu/kT}-1}\right)\left(\frac{R_2}{R_1}\right)=\frac{8\pi h\nu^3}{c^3}$$

$$\frac{h\nu}{kT}=\frac{hc}{\lambda kT}=\ln\left(\frac{R_2}{R_1}+1\right)$$

$$\lambda=\frac{hc}{kT\ln\left(\frac{R_2}{R_1}+1\right)}$$

Principle of LASER

Population inversion $\rightarrow$ making higher levels more populated than the lower levels

$$\frac{\text{rate of stimulated}}{\text{rate of spontaneous}} > 1$$

$$\frac{B\,N_2\,U_\nu}{A\,N_2}=\frac{U_\nu}{A/B}=\frac{\frac{8\pi h\nu^3}{c^3}\left(\frac{1}{e^{h\nu/kT}-1}\right)}{\frac{8\pi h\nu^3}{c^3}}$$

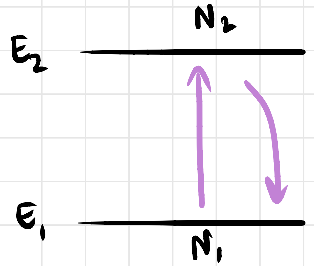
$$\approx e^{-h\nu/kT} > 1 \Rightarrow N_2 > N_1$$

Acheive by pumping (providing external energy)

Pumping Mechanisms

1. Optical - solid-state lasers (ruby laser)
2. Electrical – gas lasers
3. Forward-biasing
4. Chemical
5. Nuclear

2-Level Laser System



Condition: $N_2 > N_1$

$$\frac{N_2}{N_1} = e^{\frac{-(E_2-E_1)}{kT}} > 1 \Rightarrow \frac{-(E_2-E_1)}{kT} > 0$$

T should be -ve ⇒ not possible

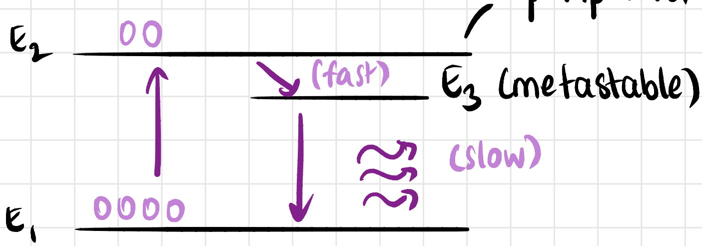
∴ not possible to construct laser with 2 levels

3-Level Laser System

One level should be metastable

Ruby laser (Al_2O_3 + Cr)

pump level (unstable: 10^{-9} s)



Al_2O_3 supports absorption

Cr supports emission

non-radiative transition

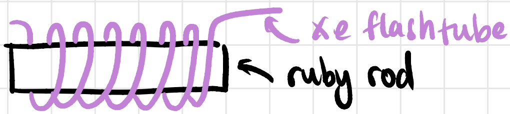
$E_2 - E_3 <<< E_2 - E_1$

Transitions from E_2 to E_3 is very fast and therefore nonradiative (generally heat)

$N_3 > N_1 \Rightarrow$ population inversion achieved

If a photon with $E = E_3 - E_1 = h\nu$ is spontaneously emitted from E_3, stimulated emission can occur (laser transition)

uses optical pumping (Xe flashtube)



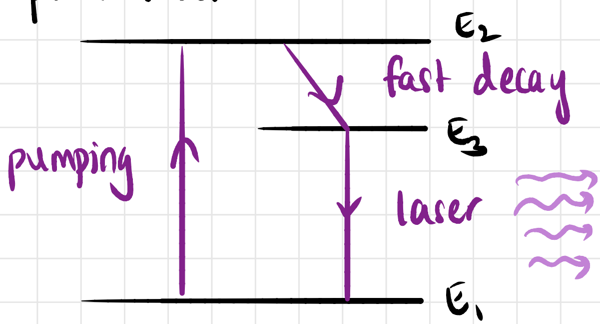
Requires heavy pumping (population inversion hard to achieve; partially decoupled system)

Ground state E_1 common to both absorption and emission processes

Ground state gets depleted quickly

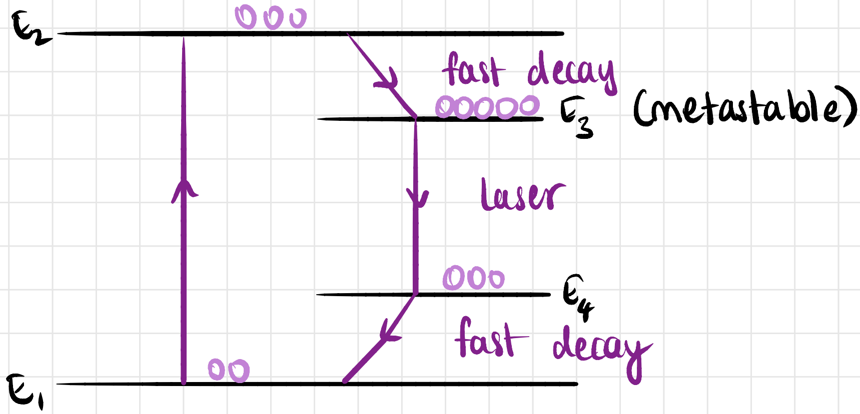
Discontinuous stimulated emission and pumping in 3-level lasers

Creates pulse laser



4-Level Laser System

Gas Laser (He-Ne, CO_2-N_2-He)



Electrical pumping (input energy is continuous)

Continuous lasers

Transition from E_2 to E_3 is non-radiative (small energy gap)

If a photon of $E = E_3 - E_4 = h\nu$ is spontaneously emitted, stimulated emission starts (laser transition)

Transition from E_4 to E_1 should be non-radiative

The absorption and emission processes are completely decoupled (more efficient laser)

N_1 replenished ⇒ allows for continuous pumping and N_3 always $> N_4$ (population inversion)

Designing a Laser

1) Active Medium
2) Pumping (external energy)
3) Resonant Cavity

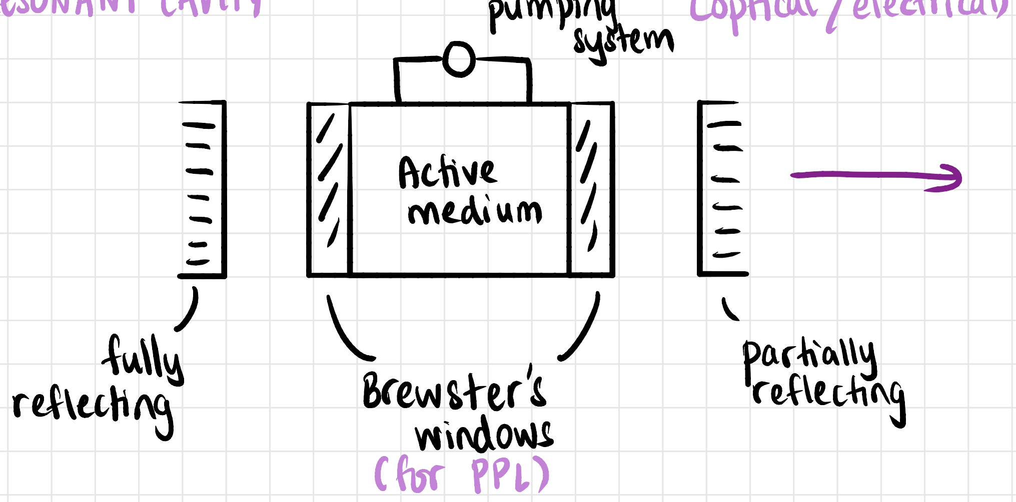
ACTIVE MEDIUM

- Consists of active material, which supports population inversion (metastable state)
- For He-Ne LASER, active species are He and Ne
- For CO_2 LASER, active species are CO_2, N_2 and He

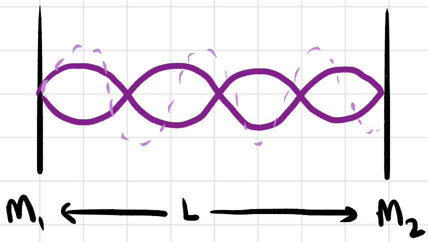
PUMPING

- Providing external energy based on type of LASER designed
- Pumping mechanisms can be electrical (gas), optical (solid state) etc.

RESONANT CAVITY



- First photon that is emitted is spontaneous (random direction)
- Pair of mirrors provide optical feedback (part of output fed as input)
- Optical feedback is necessary for sustained, amplified stimulated emission (gain $\propto e^{\ell}$ where ℓ is the distance travelled by the photon)
- Only harmonic waves can maintain constant phase (others die out)



$$L = \frac{n\lambda}{2}$$ ← resonant condition

n = longitudinal mode number

- Length of cavity should be properly designed

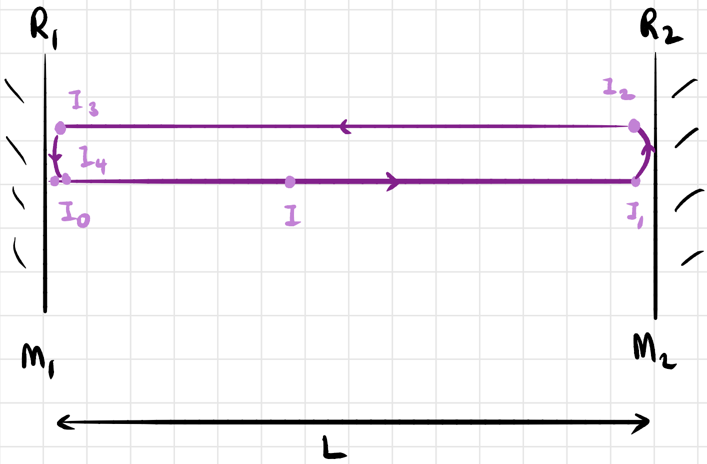
Losses in Laser Beam Intensity

1. Reflection at the mirror
   Al reflectivity ~85%

2. Absorption / scattering due to impurities

Threshold Round Trip Gain

Minimum gain for constant output



R_1 & R_2 are reflectivities of mirrors M_1 & M_2

Intensity of a beam

$$\frac{dI}{d\ell} \propto I$$

(more the no. of photons, more the rate of stimulated emission)

solving

$$I = I_0 e^{(g-\alpha)\ell}$$

g: gain coefficient
α: loss coefficient
ℓ: distance travelled

$I_0 \longrightarrow I_1$ trip (0 to L):

$$I_1 = I_0 e^{(g-\alpha)L}$$

$I_1 \longrightarrow I_2$ trip (reflection at M_2):

$$I_2 = R_2 I_1$$

$$= R_2 I_0 e^{(g-\alpha)L}$$

$I_2 \longrightarrow I_3$ trip (L to 0):

$$I_3 = I_2 e^{(g-\alpha)L}$$

$$= R_2 I_0 e^{(g-\alpha)2L}$$

$I_3 \longrightarrow I_4$ (reflection at M_1):

$$I_4 = R_1 I_3$$

$$= R_1 R_2 I_0 e^{(g-\alpha)2L}$$

For Gain

$$\frac{\text{output}}{\text{input}} = \frac{I_4}{I_0} \geq 1$$

For Minimum (Threshold) Gain

$$\frac{I_4}{I_0} = 1 \qquad \text{(constant o/p)}$$

$$\frac{\cancel{I_0} R_1 R_2 e^{(g_{th}-\alpha)2L}}{\cancel{I_0}} = 1$$

$$e^{(g_{th}-\alpha)2L} = \frac{1}{R_1 R_2}$$

$$(g_{th}-\alpha)2L = -\ln(R_1 R_2)$$

$$g_{th}-\alpha = -\frac{\ln(R_1 R_2)}{2L}$$

$$g_{th} = \alpha - \frac{\ln(R_1 R_2)}{2L}$$

If $g \geq g_{th}$, LASER formed
$g < g_{th}$, LASER dies

Q: The ratio of populations of 2 energy levels is 1.5×10^{-30}. The upper level corresponds to metastable state. Find λ of light emitted at 330 K.

$$\frac{N_1}{N_2} = e^{\frac{h\nu}{kT}}$$

$$\frac{N_2}{N_1} = e^{\frac{-h\nu}{kT}} = e^{\frac{-hc}{kT\lambda}}$$

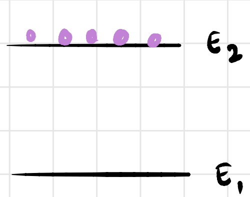
$$-\ln(1.5 \times 10^{-30}) = \frac{hc}{kT\lambda}$$

$\lambda = 635$ nm

Properties of LASER

* Properties of a photon
  - wavelength
  - phase
  - direction

1. Highly monochromatic (wavelength)



$\tau \sim 10^{-3}\,s$
(metastable)

Uncertainty in time spent by e^- in metastable state is in the order of τ (relaxation time)

$$\Delta t \sim \tau$$

uncertainty in energy of photon → $$\Delta E \sim \frac{\hbar}{2\Delta t} = \frac{\hbar}{2\tau}$$

In a spontaneously emitting system

$$\tau \sim 10^{-9}\,s$$
$$\Delta t \sim 10^{-9}\,s$$

$$\Delta E \sim \frac{\hbar}{2\tau}$$

$$|\Delta E| = \left|\Delta\left(\frac{hc}{\lambda}\right)\right| = \left|\frac{hc}{\lambda^2}\right| \Delta\lambda$$

$$\frac{h}{4\pi} \times 10^{9} = \left|\frac{hc}{\lambda^2}\right| \Delta\lambda_{st}$$

$$\Delta\lambda_{st} = \frac{\lambda^2}{4\pi c} \times 10^{9}$$

In a stimulated emission system

$$\tau \sim 10^{-3}\,s$$
$$\Delta t \sim 10^{-3}\,s$$

$$\Delta E \sim \frac{h}{4\pi\tau} = \frac{h}{4\pi} \times 10^{3}$$

$$\Delta E = \left|\frac{hc}{\lambda^2}\right| \Delta\lambda_{sp}$$

$$\Delta\lambda_{sp} = \frac{\lambda^2}{4\pi c} \times 10^{3}$$

$$\Delta\nu = \frac{c}{\lambda^2} \Delta\lambda$$

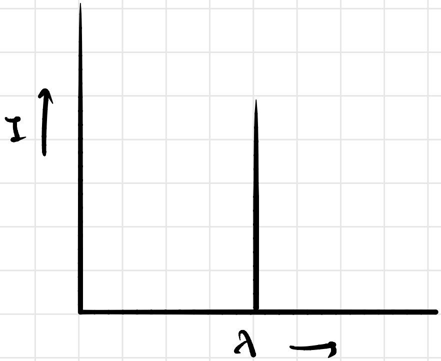
Ratio of $\Delta\lambda_{sp}$ to $\Delta\lambda_{st}$

$$\frac{\Delta\lambda_{sp}}{\Delta\lambda_{st}} \sim 10^{6}$$

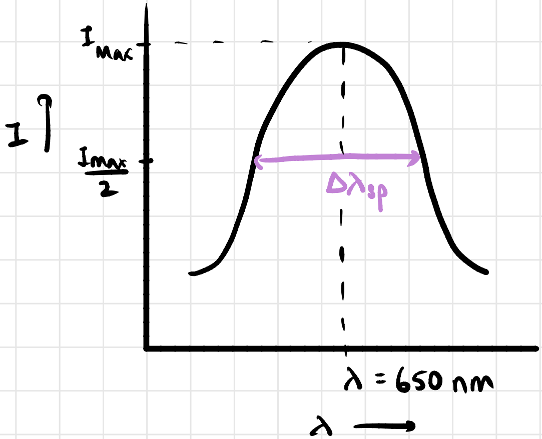
$$\Delta\lambda_{st} \sim 10^{-6}\,\Delta\lambda_{sp}$$

The spread in λ due to stimulated photon is at least a million times smaller than that of a spontaneously emitted photon.

Purely monochromatic (theory)



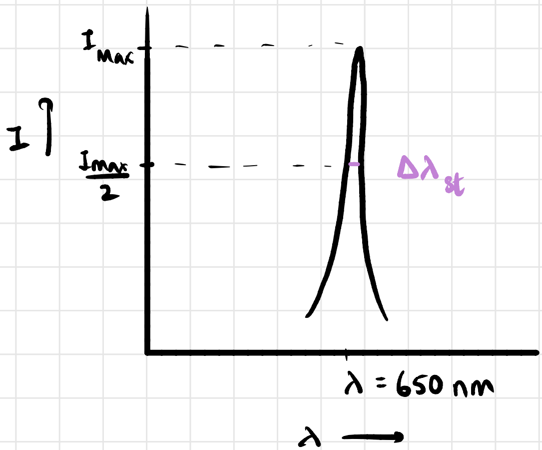
Red LED (spontaneous)



Full Width at Half Maximum

$\Delta\lambda_{sp} \sim 1 \text{Å}$

Red LASER (stimulated)



FWHM

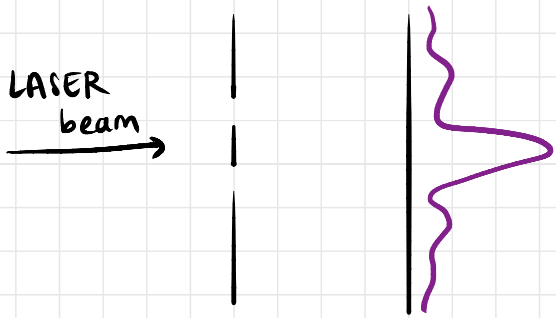
$\Delta\lambda_{st} \sim 10^{-6} \Delta\lambda_{sp}$

- $\Delta\lambda_{st} \neq 0$; there is a finite line width
- LASER systems are highly monochromatic
- No emitting process is truly monochromatic (for more than a single photon)

Reasons for Spread in Wavelength

1. Uncertainty principle
2. Spectral broadening due to Doppler effect
    - sources are moving
    - movement of atoms and molecules inside the cavity
    - instantaneous T of gas molecules could be very high
3. Energies of transitions not fully discrete; small bands

2. High coherence (phase correlation between photons)



interference pattern:
information on phase

- if interference pattern is well-defined (sharp dark fringes), phase correlation is good (coherence)

(a) Temporal Coherence

- phase is periodic at the same point

$$y(x, t_0 + T) - y(x, t_0) = \text{constant}$$

- correlation between phase at one time and phase at another time for the same point (constant)

- source not 100% monochromatic; there is a limit to temporal coherence

spread in frequency of photon

- Coherence time $\tau_c = \frac{1}{\Delta\nu}$ $\left(\Delta\nu = \frac{c}{\lambda^2} \Delta\lambda \right)$

- For truly monochromatic sources, $\tau_c = \infty$ as $\Delta\nu = 0$ (phase correlation holds true for an infinite amount of time)

- If the spread in ν is more, a common period can be found only for a short amount of time

- As time increases, phase diff. changes

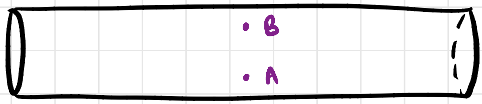
- Coherence length: largest distance for which interference is well-defined

$$l_c = \tau_c c$$

- Few kms for LASERs (τ_c is μs)

(b) Spatial Coherence

- Phase difference between two points in space of a wave front is constant over any time t



phase diff b/w A & B

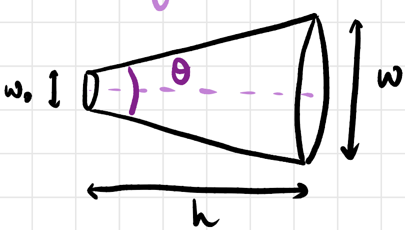
- Two different beams from different atom sources are spatially incoherent
- Limit of AB (max) → coherence width

$$w_c \approx \frac{\lambda}{\pi w_0} \qquad l_w \approx \frac{\lambda}{\theta}$$

(w_0 ← spot size)

- Highly coherent source
- For holograms, interference patterns used
- Interference patterns used for encoding information

3. Directionality



θ order of milliradians

$$\tan \theta/2 = \frac{w}{2h} \approx \frac{\theta}{2}$$

$$\theta = \frac{w}{h}$$

$$\theta = \frac{\lambda}{\pi w_0}$$

4. Intensity

- contribution of monochromaticity, coherence and low divergence
- high intensity beam for low power
- 5 mW laser over diameter of 1 mm is comparable to sunlight (should not directly view LASER)
- look up Q-switched lasers

TYPES OF LASERS

1. Atomic LASER
   transitions between e^- energy levels

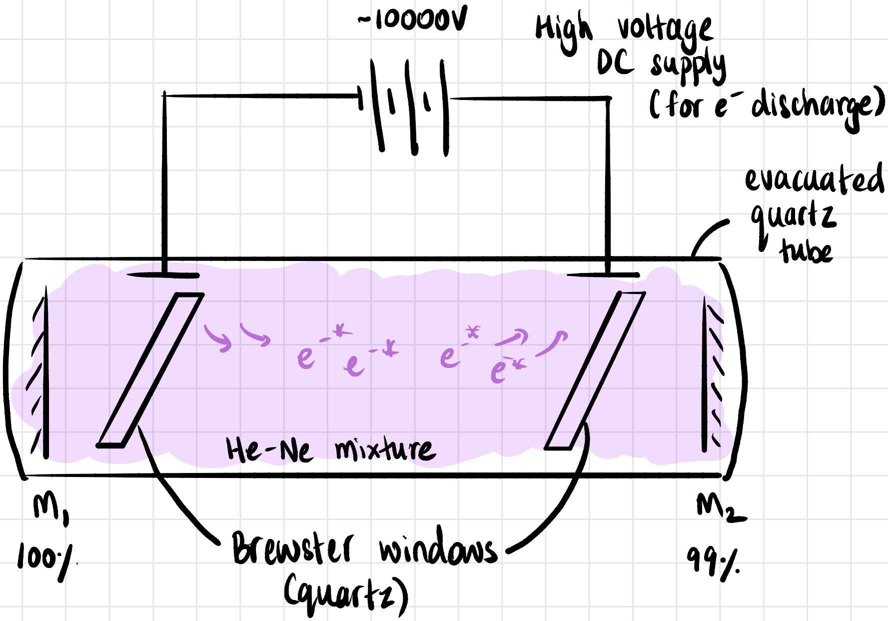
2. Molecular LASER
   transitions between molecular energy states

3. Semiconductor LASER
   transitions between VB and CB

Atomic LASER — He-Ne LASER system

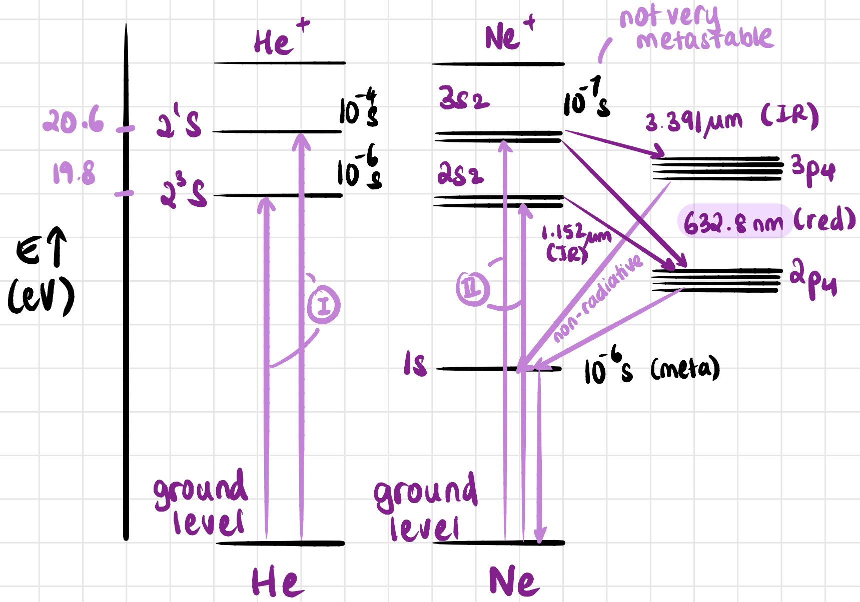
- Second LASER ever built (first - Ruby)
- Emission: 632.8 nm (red)
- Four-level laser, continuous

Construction



- Evacuated glass tube
- 1 torr pressure
- He : Ne = 10 : 1 partial pressure ⇒ 10 : 1 ratio of atoms
- Brewster windows: polarise and absorb IR
- Fast-moving e^- in gas
- Pumping mechanism: electron discharge
- ~10 mW power

Energy Level Diagram



- Energy levels are e^- levels
- Any state with $\tau > 10^{-8}$ s is metastable

Collision of (I) kind

$$e^{-*} + He \longrightarrow He^* + e^-$$

e^{-*}: excited e^-

He: more likely than Ne

Collision of (II) kind

$$He^* + Ne \longrightarrow Ne^* + He$$

He^*: excited He

- Energies of 2s and 3s energy states in Ne very close to energies of 2^3S and 2^1S states of He
- He atoms excited so that Ne higher states can be populated
- For red laser, $3s_2 \longrightarrow 2p_4$ of Ne (632.8 nm)
- $3s_2 \longrightarrow 3p_4$ is strongest transition (3.391μm most seen) and $2s_2 \longrightarrow 2p_4$ strong (1.152μm)
- Brewster windows absorbs some IR (reduction in output by 40-50%)
- CH_4 gas absorbs more IR (added in small amounts)
- To depopulate 1s quickly, tube is made narrow to increase probability of collision with walls of tube
- Air cooling system; no need water

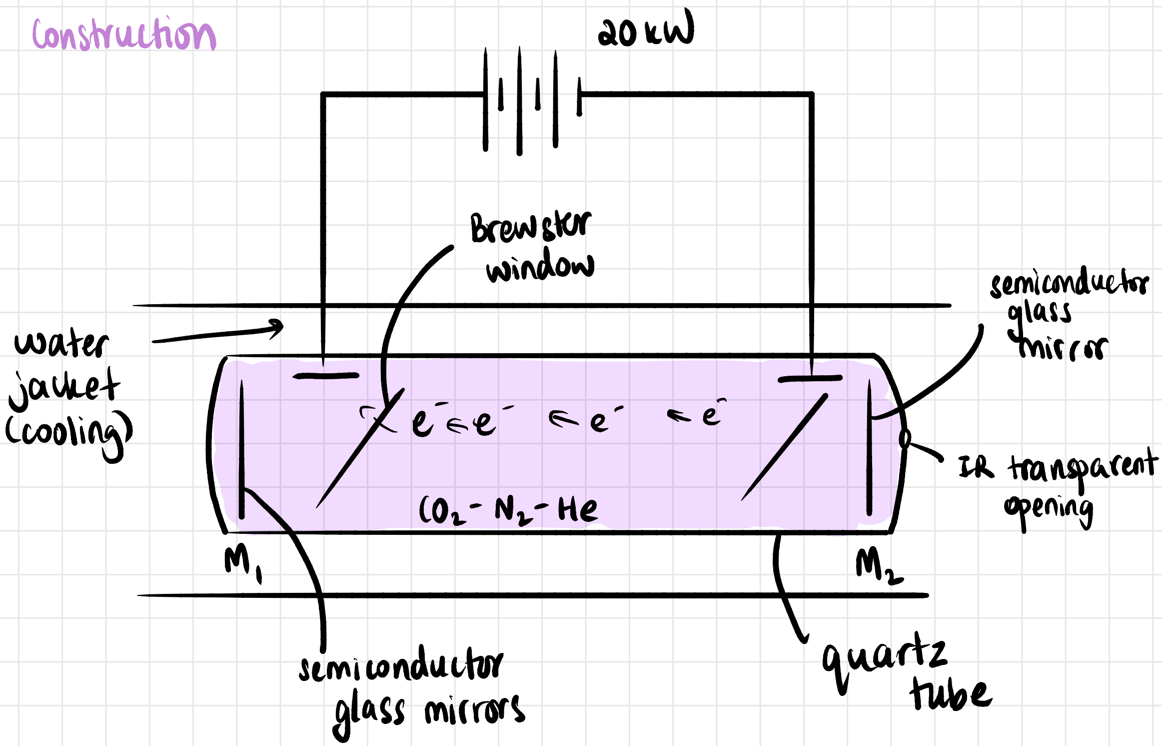
Why is He Added?

- Ne is active species, not He
- He is added to act as buffer (2^1S and 2^3S act as virtual metastable states for population inversion in Ne)
- If e^{-*} collide with Ne, most favourable transition is to 1s, not 2s and 3s
- Therefore, collision of II kind required

Molecular LASER- CO_2 LASER system

- very powerful laser (can cut through steel)
- transitions between molecular vibrational states of a molecule
- from few W to kW (used in heavy-energy industries)

Construction



- Partial pressure of N_2 is high
- N_2 & He are buffer gases
- CO_2 is active material
- High voltage supply
- N_2 in abundance
- water as coolant
- IR laser
- High power industrial lasers

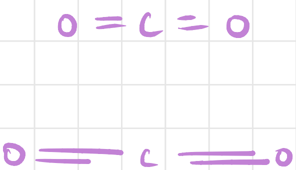
$O=C=O$

Vibrational states (linear molecule) 

3-mass, 2-spring system (molecular spectroscopy)

3 Types of Vibrating Modes

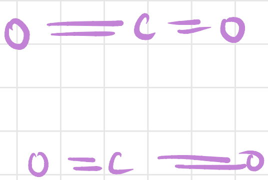
Symmetric stretch



Purely symmetric

100 I excited state
200 II excited state
⋮
n00

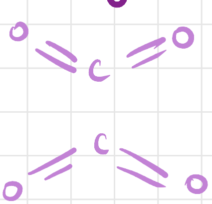
Asymmetric strech



Purely Asymmetric

001 I excited state
002 II excited state
⋮
00n

Bending

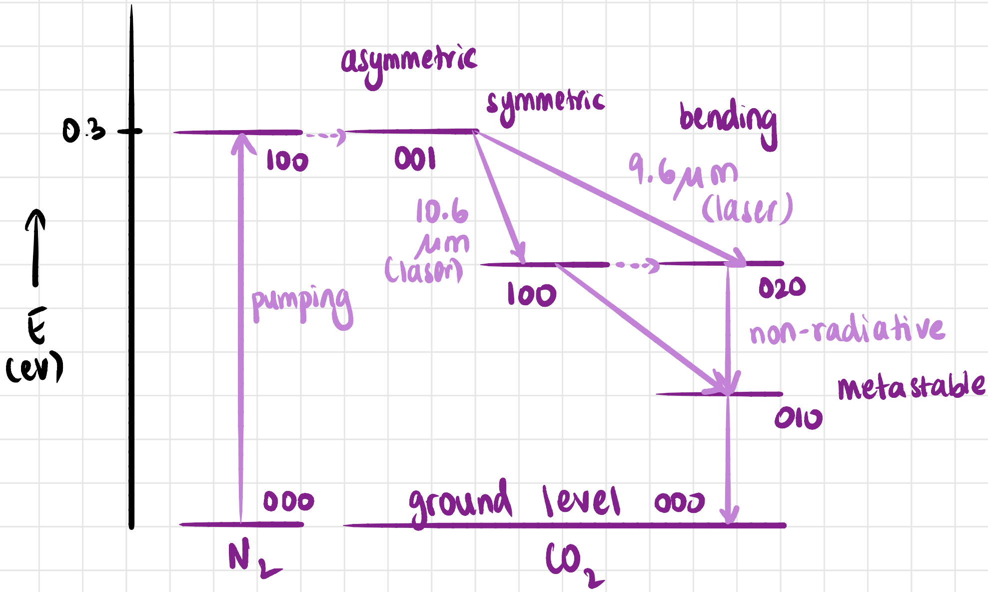


Purely Bending

010 I excited state
020 II excited state
⋮
0n0

Energy Level Diagram

(quantised energies)



Collision (I)

$$e^{-*} + \underset{(000)}{N_2} \longrightarrow \underset{(100)}{N_2^*} + e^-$$

Collision (II)

$$\underset{(100)}{N_2^*} + \underset{(000)}{CO_2} \longrightarrow \underset{(001)}{CO_2^*} + \underset{(000)}{N_2}$$

- Vibrational states: all have higher lifetimes ($\sim 10^{-3}$ s)
- N_2 used for virtual metastable states

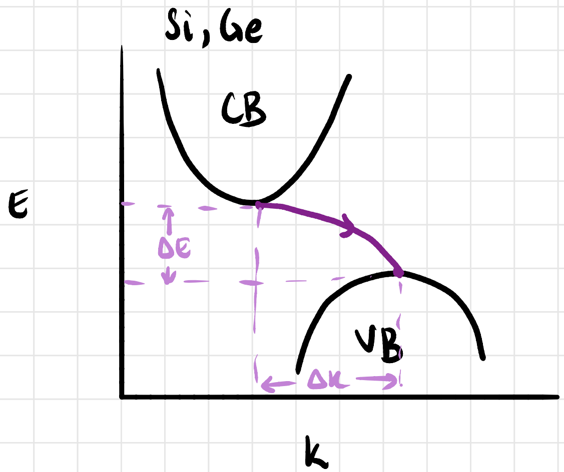
- 001 of CO_2 similar to 100 of N_2
- 10.6μm main emission
- He used to de-excite 010 state of CO_2 as 010 is unfortunately metastable
- CO_2 molecules collide with He atoms to go from 010 (bending) to 000 and increase KE of He
- Lots of heat released; water for cooling
- Ratio of $N_2 : CO_2 \approx 2:1$ (more N_2)
- Mirrors have to withstand high temp; made of semiconductors (ZnSe, Au) and expensive
- Relevant for industry

Semiconductor LASERs

- Very efficient - low power
- Beam quality not great

- Si, Ge cannot be used (indirect band gap SC)

Indirect Band Gap SC

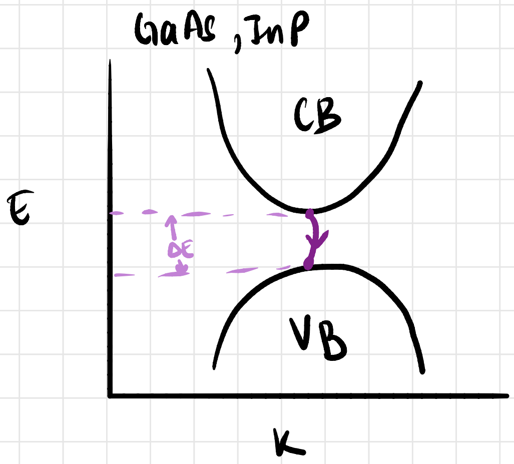


light emission not possible as there is a large Δk that cannot be carried by photons.

happens through collisions (ΔE and Δk)

band gap not in visible range

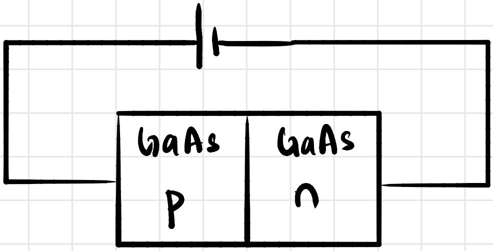
Direct Band Gap SC

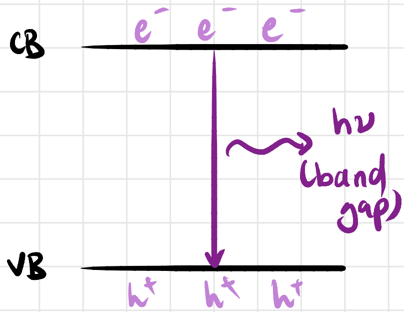


Δk is small

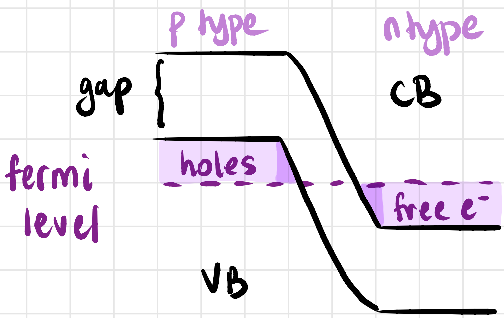
transition can give out radiation

(I) Homo Junction LASER





Heavily Doped



1. Active material

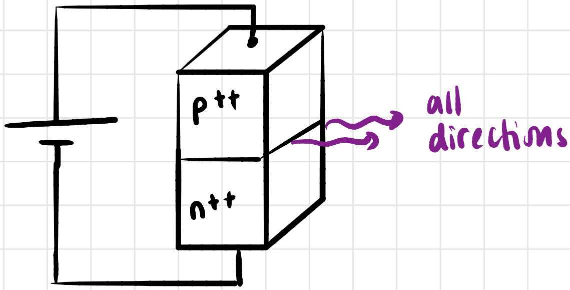
- GaAs p-n
- Heavily doped sc diode ⇒ thin depletion region
- Fermi level of n type is in CB and Fermi level of p type in VB
- Spontaneous emission (LED) at low currents and stimulated at excessive FB current

2. Energy pump

excessive FB current

3. Cavity

- needs mirrors



- SC properly cleaved in direction → reflectivity of crystal
- front & back reflective, others rough
- $L = \frac{n\lambda}{2}$

Operating conditions

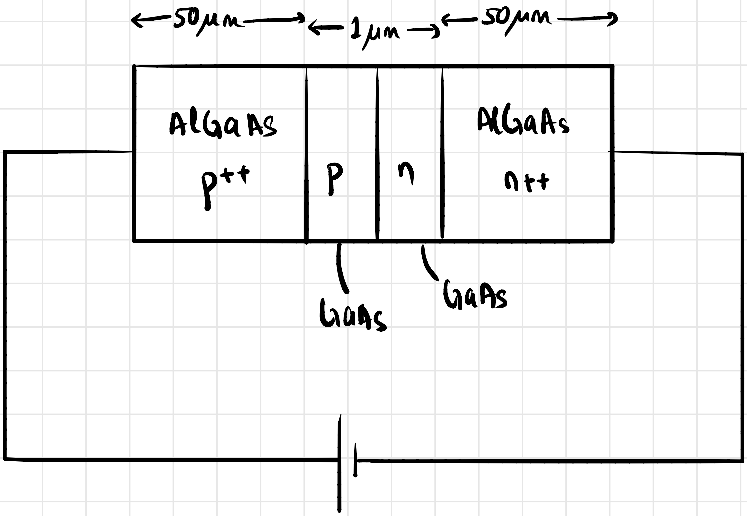
- Very high I required
- Very low temperatures
- At T=40K, I=10A, 10 μJ LASER
- Not very practical

Drawbacks

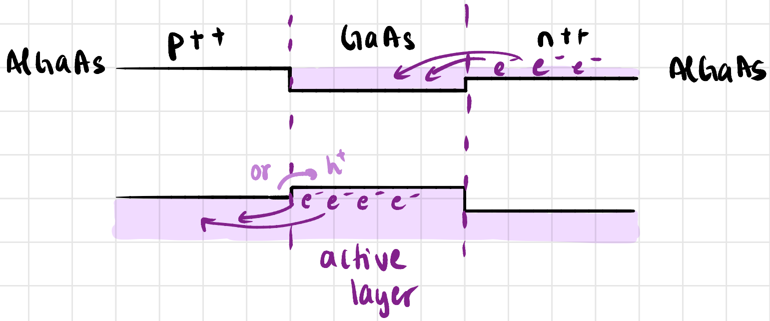
- e^-, h^+ conc. in active layer is very low
- photons lost; all directions

(II) Hetero Junction LASER

- fixed problems of homo
- multilayered heterojunction (many layers)
- AlGaAs — doped GaAs with Al (higher E_g)
- Al doped at Ga sites



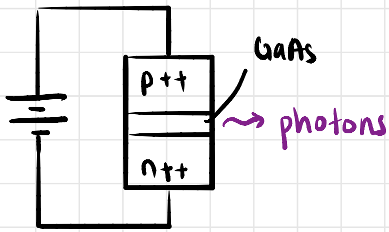
Energy Band diagram



- GaAs has lower band gap
- e^- in CB of n and h^+ in VB of p

1. Charge confinement

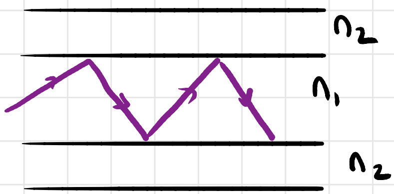
- In normal SC diode, charges are diffused and recombination not necessarily achieved
- Artificial population inversion in the active layer
- High concentrations of e^- and h^+ in active layer, allowing for recombination in FB and stimulated photon emission



2. Photon confinement

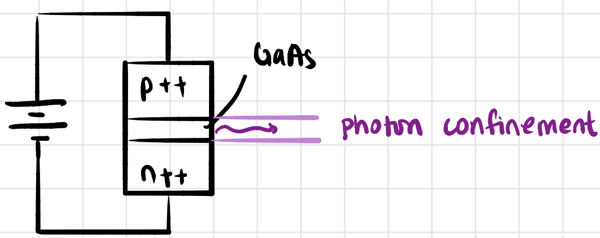
- AlGaAs has a lower refractive index than GaAs

total internal reflection



if $n_1 > n_2$

- Similar kind of TIR occurs in Hetero junction LASER; layer in which all photons are going to be contained



Operating conditions

- room temperature
- 1500 A cm^{-2} to 600 A cm^{-2}
- 5mW - 10mW LASER system

Q: The ratio of populations of upper excited state to lower energy state of a system at 300K is found to be 1.2×10^{-19}. Find λ of radiation emitted and energy density.

$$\frac{N_1}{N_2} = e^{\frac{h\nu}{kT}}$$

$$\frac{N_2}{N_1} = e^{\frac{-hc}{\lambda kT}}$$

$$-\ln(1.2 \times 10^{-19}) = \frac{hc}{\lambda k(300)}$$

$1.248377792 \times 10^{-14}$

A/B

$$\lambda = 1.10 \mu m$$

$$U_\nu = \frac{8\pi h\nu^3}{c^3}\left(\frac{1}{e^{\frac{h\nu}{kT}} - 1}\right)$$

$$U_\nu = \frac{8\pi h}{\lambda^3}\left(\frac{1}{\frac{1}{1.2\times10^{-19}} - 1}\right)$$

$\lambda = \frac{c}{\nu}$

$$U_\nu = 1.498 \times 10^{-33}\ J s m^{-3}$$

Q: A laser emission from a certain laser has an output power of 10mW. $\lambda = 632.8$ nm, find rate of emission of stimulated photons.

$$\text{power} = \text{rate of emission} \times \frac{hc}{\lambda}$$

↙ energy per emission

↗ emissions per sec

$$10 \times 10^{-3} = r \times \frac{hc}{632.8\text{nm}}$$

$$r = 3.19 \times 10^{16}\ s^{-1}$$

Q: A pulsed laser has a power of 1mW and lasts for 10 ns. If no. of photons emitted is 3.491×10^7, $\lambda = ?$

$$\text{power} = \text{rate} \times \frac{hc}{\lambda}$$

$$10^{-3} = \frac{3.491 \times hc \times 10^7}{\lambda \times 10 \times 10^{-9}}$$

$$\lambda = 693 \text{ nm}$$

Q: Find the ratio of the rate of stimulated emission to the rate of spontaneous emission for a system emitting a wavelength of 632.8 nm at 300K.

R_1 = rate of stimulated
R_2 = rate of spontaneous
$\lambda = 632.8\ nm$
$T = 300\ K$

$$R_1 = B N_2 U_\nu \Rightarrow B = \frac{R_1}{N_2 U_\nu}$$
$$R_2 = A N_2 \Rightarrow A = \frac{R_2}{N_2}$$

$$\frac{A}{B} = \frac{8\pi h \nu^3}{c^3} = \frac{8\pi h}{\lambda^3}$$

$$\frac{\left(\frac{R_2}{N_2}\right)}{\left(\frac{R_1}{N_2 U_\nu}\right)} = \frac{8\pi h}{\lambda^3}$$

$$U_\nu \frac{R_2}{R_1} = \frac{8\pi h}{\lambda^3}$$

$$U_\nu = \frac{8\pi h}{\lambda^3}\left(\frac{R_1}{R_2}\right)$$

$$U_\nu = \frac{8\pi h}{\lambda^3}\left(\frac{1}{e^{h\nu/kT} - 1}\right)$$

$$\frac{R_1}{R_2} = \frac{1}{e^{h\nu/kT} - 1} = \frac{1}{e^{hc/\lambda kT} - 1}$$

$$\frac{R_1}{R_2} = 1.22 \times 10^{-33}$$

Q. $B_{10} = 2.7 \times 10^{19}\ m^3/W\text{-}s^3$ for a particular atom, find the lifetime of the 1 to 0 transition at
(a) 550 nm (b) 55 nm

$$\text{rate of emission} = B_{10} N_2 U_\nu$$

$$\tau \approx \frac{1}{A}$$

$$\frac{A_{10}}{B_{10}} = \frac{8\pi h}{\lambda^3}$$

(i) $A_{10} = 2.7 \times 10^6\ m^3/W\text{-}s^3$

$$\tau = \frac{1}{A} = 3.7 \times 10^{-7}\ s$$

$$\tau = 370\ ns$$

(ii) $A_{10} = 2.7 \times 10^9$

$$\tau = 3.7 \times 10^{-10} = 0.37\ ns$$

Q: The energy levels in a 2-level atom are separated by 2eV. There are 3×10^{18} atoms in the upper level and 1.7×10^{18} atoms in the lower level. Coefficient of stimulated emission is $3.2 \times 10^5\ m^3/W s^3$ and the spectral radiance is $4\ Wm^{-2}Hz$. Calculate rate of stimulated emission.

$N_2 = 3 \times 10^{18}$ $N_1 = 1.7 \times 10^{18}$ $h\nu = 2\ eV$

$U_\nu = 4\ Wm^{-2}s^{-1}$ $B = 3.2 \times 10^5\ m^3/Ws^3$

rate of stimulated = $B N_2 U_2$

Q: There is emission of $\lambda = 546$ nm, $\Delta\lambda = 0.1$ nm. Find A.

$$A = \frac{1}{\tau} \qquad \tau = \frac{1}{\Delta\nu}$$

$$E = \frac{hc}{\lambda} = h\nu$$

$$\Delta\nu = \frac{c}{\lambda^2}\Delta\lambda$$

$$\Delta\nu = 1 \times 10^{11}$$

$$\tau_c = 9.94 \times 10^{-12}$$

$$A = 1 \times 10^{11}$$